6.6-2

From the column load tables with $KL = 1.0(13.5) = 13.5$ ft, $\phi_c P_n = 723$ kips

$$\frac{P_u}{\phi_c P_n} = \frac{140}{723} = 0.1936 < 0.2 \quad \therefore \text{ use AISC Equation H1-1b.}$$

For the axis of bending, $\dfrac{KL}{r} = \dfrac{K_x L}{r_x} = \dfrac{0.8(13.5)(12)}{4.49} = 28.86$

$$P_{e1} = \frac{\pi^2 EA_g}{(KL/r)^2} = \frac{\pi^2 (29,000)(22.6)}{(28.86)^2} = 7766 \text{ kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4(0) = 0.6$$

$$B_1 = \frac{C_m}{1-\frac{P_u}{P_{e1}}} = \frac{0.6}{1-\frac{140}{7766}} = 0.611 < 1.0 \quad \text{use } B_1 = 1.0$$

From the beam design charts with $L_b = 13.5$ ft, $\phi_b M_n = 353$ ft-kips for $C_b = 1.0$

For the end conditions and loading of this problem, $C_b = 1.67$ (Figure 5.15g, textbook).

For $C_b = 1.67$, $\phi_b M_n = 1.67(353) = 593$ ft-kips > $\phi_b M_p = 366$ ft-kips

$\therefore$ use $\phi_b M_n = \phi_b M_p = 366$ ft-kips

AISC Equation H1-1b:

$$\frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = \frac{140}{2(723)} + \left(\frac{M_{ux}}{366} + 0\right) \leq 1.0$$

$M_{ux} \leq 330.6$ ft-kips (amplified moment)

Compute the unamplified moment:

$$M_{ux} = B_1 M_{ntx} + B_2 M_{\ell tx} = 1.0 M_{ntx} + 0 = 330.6 \text{ ft - kips}$$

$M_{ntx} \leq 331$ ft-kips

$M_{ntx} = 331$ ft - kips

6.6-9

Compute the compressive design strength:

$$\frac{KL}{r_x} = \frac{20(12)}{5.83} = 41.17, \quad \frac{KL}{r_y} = \frac{10(12)}{1.53} = 78.43 \text{ (controls)}$$

$$\lambda_c = \frac{KL}{r\pi}\sqrt{\frac{F_y}{E}} = \frac{78.43}{\pi}\sqrt{\frac{36}{29,000}} = 0.8796 < 1.5 \quad \therefore \text{ use AISC Eq. E2-2:}$$

$$F_{cr} = (0.658)^{\lambda_c^2} F_y = (0.658)^{(0.8796)^2}(36) = 26.04 \text{ ksi}$$

$$\phi_c P_n = \phi_c A_g F_{cr} = 0.85(10.0)(26.04) = 221.4 \text{ kips}$$

Determine which interaction equation to use:

$$\frac{P_u}{\phi_c P_n} = \frac{15}{221.4} = 0.06775 < 0.2 \quad \therefore \text{ use AISC Equation H1-1b.}$$

For the axis of bending, $\frac{KL}{r} = \frac{K_x L}{r_x} = 41.17$

$$P_{e1} = \frac{\pi^2 E A_g}{(KL/r)^2} = \frac{\pi^2 (29,000)(10.0)}{(41.17)^2} = 1689 \text{ kips}$$

For transversely loaded members with unrestrained ends, $C_m = 1.0$

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_{e1}}} = \frac{1.0}{1 - \frac{15}{1689}} = 1.009$$

$$M_{nt} = \frac{1}{8} w_u L^2 = \frac{1}{8}(2.9)(20)^2 = 145.0 \text{ ft-kips}$$

$$M_u = B_1 M_{nt} + B_2 M_{lt} = 1.009(145.0) + 0 = 146.3 \text{ ft-kips}$$

From the beam design charts with $L_b = 10$ ft, $\phi_b M_n = 132$ ft-kips for $C_b = 1.0$

For the end conditions and loading of this problem, $C_b = 1.30$ (Figure 5.15b, textbook).

For $C_b = 1.30$, $\phi_b M_n = 1.30(132) = 171.6$ ft-kips $> \phi_b M_p = 147$ ft-kips

$\therefore$ use $\phi_b M_n = \phi_b M_p = 147$ ft-kips

AISC Equation H1-1b:

$$\frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = \frac{0.06775}{2} + \left(\frac{146.3}{147} + 0\right) = 1.03 > 1.0 \quad \text{(N.G.)}$$

<u>Member is unsatisfactory</u>

6.7-1

Determine the axial compressive design strength:

$$\frac{K_x L}{r_x/r_y} = \frac{1.7(14)}{1.67} = 14.25 \text{ ft} > K_y L = 14 \text{ ft} \quad \therefore \text{ use } KL = 14.25 \text{ ft}$$

From the column load tables with $KL = 14.25$ ft, $\phi_c P_n = 1165$ kips.

$$\frac{P_u}{\phi_c P_n} = \frac{300}{1165} = 0.2575 > 0.2 \quad \therefore \text{ use AISC Equation H1-1a.}$$

Braced condition:

For the axis of bending, $\frac{KL}{r} = \frac{K_x L}{r_x} = \frac{1.0(14)(12)}{6.22} = 27.01$

$$P_{e1} = \frac{\pi^2 E A_g}{(KL/r)^2} = \frac{\pi^2 (29{,}000)(32.0)}{(27.01)^2} = 12{,}550 \text{ kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{35}{62}\right) = 0.3742$$

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_{e1}}} = \frac{0.3742}{1 - \frac{300}{12{,}550}} = 0.383 < 1.0 \quad \therefore \text{ use } B_1 = 1.0$$

Sway condition:

$$B_2 = \frac{1}{1 - \frac{\sum P_u}{\sum P_{e2}}} = \frac{1}{1 - \frac{8200}{54{,}000}} = 1.179$$

The total amplified moment is

$$M_u = B_1 M_{nt} + B_2 M_{lt} = 1.0(35) + 1.179(135) = 194.2 \text{ ft-kips}$$

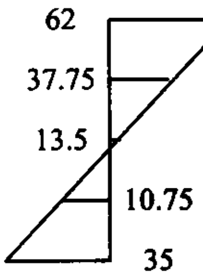
or $M_u = 1.0(62) + 1.179(54) = 125.7$ ft-kips < 194.2 ft-kips

Use $M_u = 194.2$ ft-kips

Compute the moment strength.

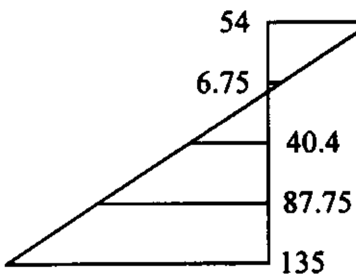
From the beam design charts with $L_b = 14$ ft, $\phi_b M_n = 715$ ft-kips for $C_b = 1.0$

Compute C_b for the nonsway case:



$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$
$$= \frac{12.5(62)}{2.5(62) + 3(37.75) + 4(13.5) + 3(10.75)} = 2.186$$

Compute C_b for the sway case:



$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$
$$= \frac{12.5(135)}{2.5(135) + 3(6.75) + 4(40.5) + 3(87.75)} = 2.155$$

Conservatively, use $C_b = 2.155$. For $C_b = 2.155$, $\phi_b M_n = 2.155(715) = 1541$ ft-kips
From the beam design charts, $\phi_b M_p = 720$ ft-kips
Since 1541 ft-kips > $\phi_b M_p$, use $\phi_b M_n = \phi_b M_p = 720$ ft-kips

AISC Equation H1-1a:

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = \frac{300}{1165} + \frac{8}{9}\left(\frac{194.2}{720} + 0\right) = 0.497 < 1.0 \quad \text{(OK)}$$

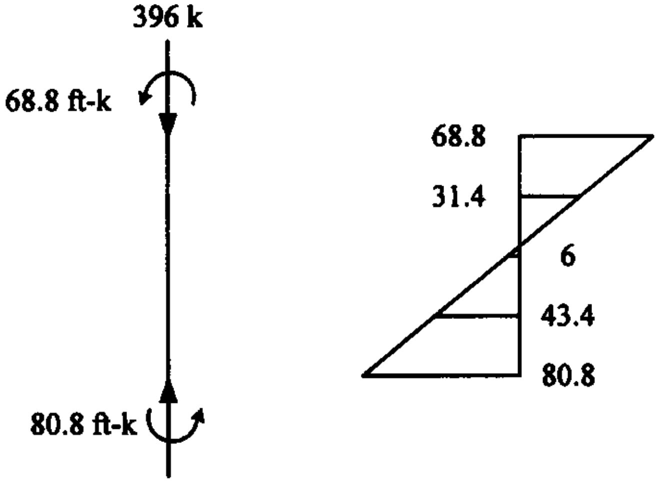
Member is satisfactory

6.7-2

AISC Load combinations A4-1, A4-3, A4-5, and A4-6 can be eliminated, leaving A4-2 and A4-4 to be considered.

Load Combination A4-2:

$$P_u = 1.2D + 1.6L = 1.2(90) + 1.6(180) = 396 \text{ kips}$$
$$M_{top} = 1.2(12) + 1.6(34) = 68.8 \text{ ft-kips}$$
$$M_{bot} = M_{nt} = 1.2(14) + 1.6(40) = 80.8 \text{ ft-kips}$$



Since the frame and loading are symmetrical, there are no sidesway moments, and

$$M_{\ell t} = 0$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{68.8}{80.8}\right) = 0.2594$$

For the axis of bending, $\frac{KL}{r} = \frac{K_x L}{r_x} = \frac{0.85(16)(12)}{5.98} = 27.29$

$$P_{e1} = \frac{\pi^2 E A_g}{(KL/r)^2} = \frac{\pi^2 (29{,}000)(17.9)}{(27.29)^2} = 6879 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_{e1}}} = \frac{0.2594}{1 - \frac{396}{6879}} = 0.2752 < 1.0 \quad \therefore \text{use } B_1 = 1.0$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(80.8) + 0 = 80.8 \text{ ft-kips}$$

From the beam design charts with $L_b = 16$ ft, $\phi_b M_n = 335$ ft-kips for $C_b = 1.0$

$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3 M_A + 4 M_B + 3 M_C}$$

$$= \frac{12.5(80.8)}{2.5(80.8) + 3(31.4) + 4(6) + 3(43.4)} = 2.242$$

For $C_b = 2.242$, $\phi_b M_n = 2.242(335) = 751.2$ ft-kips

From the beam design charts, $\phi_b M_p = 382.5$ ft-kips

Since 751.2 ft-kips > $\phi_b M_p$, use $\phi_b M_n = \phi_b M_p = 382.5$ ft-kips

Determine the critical axis for axial compressive strength:

$$\frac{K_x L}{r_x / r_y} = \frac{0.85(16)}{2.44} = 5.57 \text{ ft} < K_y L = 16 \text{ ft}$$

From the column load tables with $KL = 16$ ft, $\phi_c P_n = 486$ kips.

$$\frac{P_u}{\phi_c P_n} = \frac{396}{486} = 0.8148 > 0.2 \quad \therefore \text{ use AISC Equation H1 - 1a.}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.8148 + \frac{8}{9}\left(\frac{80.8}{382.5} + 0\right) = 1.00 \quad \text{(OK)}$$

Load combination A4-4:

$$P_u = 1.2D + 1.3W + 0.5L = 1.2(90) + 1.3(0) + 0.5(180) = 198 \text{ kips}$$
$$M_{nt} = 1.2(14) + 0.5(40) = 36.8 \text{ ft-kips}$$
$$M_{lt} = 1.3W = 1.3(100) = 130 \text{ ft-kips}$$

For the braced condition, $M_{top} = 1.2(12) + 0.5(34) = 31.4$ ft-kips $\therefore M_1 = 31.4$ ft-kips and $M_2 = 36.8$ ft-kips.

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{31.4}{36.8}\right) = 0.2587$$

$$B_1 = \frac{C_m}{1 - \dfrac{P_u}{P_{e1}}} = \frac{0.2587}{1 - \dfrac{198}{6879}} = 0.2664 < 1.0 \quad \therefore \text{ use } B_1 = 1.0$$

Compute amplification factor for sidesway moment.

$$\frac{KL}{r} = \frac{K_x L}{r_x} = \frac{1.2(16)(12)}{5.98} = 38.53$$

$$P_{e2} = \frac{\pi^2 E A_g}{(KL/r)^2} = \frac{\pi^2 (29{,}000)(17.9)}{(38.53)^2} = 3451 \text{ kips}$$

If we assume that $\dfrac{\sum P_u}{\sum P_{e2}} = \dfrac{P_u}{P_{e2}}$, then $B_2 = \dfrac{1}{1 - \dfrac{P_u}{P_{e2}}} = \dfrac{1}{1 - \dfrac{198}{3451}} = 1.061$

The total amplified moment is

$$M_u = B_1 M_{nt} + B_2 M_{lt} = 1.0(36.8) + 1.061(130) = 174.7 \text{ ft - kips}$$

From earlier computations, $\phi_b M_n = 382.5$ ft-kips

Determine the critical axis for axial compressive strength:

$$\frac{K_x L}{r_x / r_y} = \frac{1.2(16)}{2.44} = 7.87 \text{ ft} < K_y L = 16 \text{ ft}$$

From the column load tables with $KL = 16$ ft, $\phi_c P_n = 486$ kips.

$$\frac{P_u}{\phi_c P_n} = \frac{198}{486} = 0.4074 > 0.2 \quad \therefore \text{ use AISC Equation H1 - 1a.}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.4074 + \frac{8}{9}\left(\frac{174.7}{382.5} + 0\right) = 0.813 < 1.0 \quad \text{(OK)}$$

(Load combination A4-2 is the worst case) Member is satisfactory